PMM U.S.S.R., Vol.49, No.3, pp.356-362, 1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

THEORY OF FAST TECTONIC WAVES*

V.N. NIKOLAYEVSKII and T.K. RAMAZANOV

A two-dimensional model is proposed for the propagation of tectonic stress waves that are the trigger of earthquakes in seismoactive regions, and are due to bending-compression of the lithospheric slab on the asthenospheric surface flow thereby neglecting inertia forces. The lithosphere is modelled by a thin elastic slab, and the asthenosphere by the flow of a highly viscous incompressible fluid. Their interaction occurs because of the presence of a vertical shift and the action of viscous tangential forces on the lithosphere-asthenosphere interface. To obtain a system of linear equations, longitudinal and transverse potentials are introduced. The periodic low-intensity waves turn out to be standing waves, although also diffusely expanding if there is just no solid-body displacement of the lithosphere on the asthenosphere. If motion of the lithosphere over the earth's crust exists, then solitary waves are possible that take their energy from the stationary asthenospheric flow.

The tectonic waves under consideration /1,2/ with characteristic periods of 2,3,6,11 years and propagation velocities of 10-100 km/yr are extremely slow compared with seismic waves, but are sufficiently rapid in the time scale of ordinary tectonic processes, comprising millions of years. The reality of the existence of such waves can be judged, for instance, from the recently detected /3/ change of the tectonic stresses (with an 11 year cycle and an amplitude of the order of 0.1 GPa) in the subductable lithospheric slabs. The characteristic shear modulus G of the lithosphere and the viscosity μ of the asthenosphere are estimated by the numbers $5\cdot 10^{10}$ Pa and 10^{20} Pa.sec, and 10^{10} Pa and 10^{18} Pa.sec, respectively; consequently, the relaxation time μ/G of processes in the lithosphere has the requisite order of from 1 - 30 years only for the "lithosphere + asthenosphere" complex. This suggests the construction of an adequate model of the process by analogy with surface waves on a moving film of fluid /4/, but by replacing the capillary layer by an elastic plate. The adequacy of such an approach was confirmed by a simple preliminary analysis /2/ of a one-dimensional process. Meanwhile, the actual processes occur along two-dimensional lithospheric slabs, which requires the construction of a two-dimensional theory as well as a more careful consideration of the forces acting on the lithospheric slab.

1. In constructing the theory we bear in mind that the width of the lithosphere $2h$ and the asthenosphere $2H$, of the order of 100 km, the velocity $v^c \sim 10$ cm/yr of the stationary flow in the asthenosphere, and the lithosphere rate of rise $\eta^{\cdot} \sim 10-100$ cm/yr, are measurable, however, for real, i.e., not ideally elastic systems.

We consider the strain of a thin elastic slab ($k=1$) floating on a viscous incompressible fluid layer ($k=2$) (see the figure). We neglect inertia forces in the momentum balance equations but we take account of the gravity forces

$$\frac{\partial \sigma_{lm}^{(k)}}{\partial x_m} = -\rho_k g_l \delta_{l3}, \quad l, m = 1, 2, 3 \tag{1.1}$$

where $\sigma_{lm}^{(k)}$ are the stress tensors, ρ_k is the density, $g_3 = g$ is the acceleration due to gravity, δ_{lm} is the unit tensor, where the x_1, x_2 axes are in the horizontal plane while the x_3 axis is directed into the depth. Summation is over the subscript m. Averaging of (1.1) over the transverse layer thickness $x_3^{(k1)} \leqslant x_3 \leqslant x_3^{(k2)}$ connects the total stresses $N_{ij}^{(k)}$ in the transverse layer sections with the contact forces

$$\frac{\partial N_{ij}^{(k)}}{\partial x_j} = \sigma_{i3}^{(k)}(x_3^{(k1)}) - \sigma_{i3}^{(k)}(x_3^{(k2)}) + \rho_k g_i \delta_{i3}(x_3^{(k1)} - x_3^{(k2)}) \tag{1.2}$$

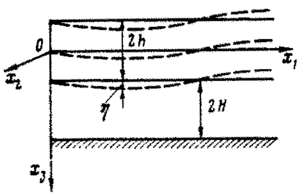
*Prikl. Matem. Mekhan., 49, 3, 462-469, 1985

$$\left(N_{ij}^{(k)} = \int\limits_{x_3^{(k1)}}^{x_3^{(k2)}} \sigma_{ij}^{(k)}\, dx_3\right)$$

Here η is the deflection of the plate middle surface $\eta \ll h \sim H \leqslant \lambda/2,\ x_3^{(11)} = -h - \eta,\ x_3^{(12)} = x_3^{(21)} = h + \eta,\ x_3^{(22)} = h + 2H,\ \lambda$ is the wavelength, $i, j = 1, 2$.

We seek the field of horizontal velocities in the liquid layer in the form

$$v_i = \left(w_i^\circ + \frac{\partial U_i}{\partial t}\right)\left(1 + \frac{h - x_3}{2H}\right) - \left(v_i^\circ + \frac{\partial \varphi_i}{\partial t}\right)\left(1 + \frac{h - x_3}{2H}\right)\frac{h - x_3}{2H} \tag{1.3}$$



where $U_i\,(x_j, h + \eta, t)$ is the non-stationary horizontal shift of the lithosphere in the plane of contact with the asthenosphere, and $\partial \varphi_i/\partial t$ is the non-stationary head velocity component due to the ascent (descent) of the lithospheric slab. The stationary $(t \geqslant 0)$ lithosphere w_i° and asthenosphere v_i° velocities satisfy the continuity equation $\partial w_i^\circ/\partial x_i = 0,\ \partial v_i^\circ/\partial x_i = 0$ in a two-dimensional plane, and are of the same order of magnitude in absolute value as the non-stationary velocities: $v^\circ \sim w^\circ \sim \partial U/\partial t \sim \partial \varphi/\partial t$.

Integration of the continuity equation $\partial v_l/\partial x_l = 0$ across the liquid layer connects the mean velocity $\langle v_i \rangle$ with the values v_3 on the horizontal boundaries

$$\frac{\partial \langle v_i \rangle}{\partial x_i} + \frac{1}{2H}\left[v_3(h + 2H) - v_3(h + \eta)\right] = 0, \tag{1.4}$$

$$\langle v_i \rangle = \frac{1}{2H}\int\limits_{h+\eta}^{h+2H} v_i\, dx_3$$

Substituting (1.3) into (1.4), taking the equalities $v_3(h + \eta) \approx \partial \eta/\partial t,\ v_3(h + 2H) = 0$ into account, we reduce (1.4) to the equation

$$\frac{\partial \eta}{\partial t} = H\frac{\partial^2 U_i}{\partial t\, \partial x_i} + \frac{H}{3}\frac{\partial^2 \varphi_i}{\partial t\, \partial x_i} \tag{1.5}$$

for the vertical displacement η on the contact between the asthenosphere and the lithosphere, neglecting second- and higher-order infinitesimal terms.

The fluid is assumed to be Newtonian, i.e.,

$$\sigma_{lm}^{(2)} = -p\delta_{lm} + \mu\left(\frac{\partial v_l}{\partial x_m} + \frac{\partial v_m}{\partial x_l}\right) \tag{1.6}$$

where p is the pressure. Because of the representation (1.3) we hence obtain expressions for the contact stresses as well as for the average stresses (with respect to the width of the asthenospheric layer)

$$\sigma_{33}^{(2)}(x_j, h + \eta, t) = -p - 2\mu\frac{\partial^2 U_i}{\partial t\, \partial x_i} \tag{1.7}$$

$$\sigma_{i3}^{(2)}(x_j, h + \eta, t) = \mu\left[\frac{\partial^2 \eta}{\partial t\, \partial x_i} + \frac{1}{2H}\left(\frac{\partial \varphi_i}{\partial t} - \frac{\partial U_i}{\partial t} + v_i^\circ - w_i^\circ\right)\right]$$

$$\sigma_{i3}^{(2)}(x_j, h + 2H, t) = -\frac{\mu}{2H}\left(\frac{\partial \varphi_i}{\partial t} + \frac{\partial U_i}{\partial t} + v_i^\circ + w_i^\circ\right)$$

$$\sigma_{ij}^{(2)}(x_j, h + \eta, t) = -p\delta_{ij} + \mu\left[\frac{\partial}{\partial t}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \left(\frac{\partial w_i^\circ}{\partial x_j} + \frac{\partial w_j^\circ}{\partial x_i}\right)\right]$$

$$\frac{N_{ij}^{(2)}}{H} = -2\langle p \rangle \delta_{ij} + \mu\left\{\frac{\partial w_i^\circ}{\partial x_j} + \frac{\partial w_j^\circ}{\partial x_i} + \frac{\partial}{\partial t}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \frac{1}{3}\left[\frac{\partial v_i^\circ}{\partial x_j} + \frac{\partial v_j^\circ}{\partial x_i} + \frac{\partial}{\partial t}\left(\frac{\partial \varphi_i}{\partial x_j} + \frac{\partial \varphi_j}{\partial x_i}\right)\right]\right\}$$

$$\left(\langle p \rangle = \frac{1}{2H}\int\limits_{h+\eta}^{h+2H} p\, dx_3\right)$$

Substituting (1.7) into (1.2) and eliminating the vertical displacement by using (1.5), we reduce the average flow equations in the asthenosphere to the simple form

$$v_i^\circ + \frac{\partial \varphi_i}{\partial t} - H^2\frac{\partial}{\partial t}\nabla^2\left(U_i + \frac{1}{3}\varphi_i\right) = -\frac{2H^2}{\mu}\frac{\partial \langle p \rangle}{\partial x_i} \tag{1.8}$$

where ∇^2 is the two-dimensional Laplace operator.

2. We shall give the displacement distribution in the lithosphere according to the Kirchhoff-Love hypothesis /5/ $U_i = u_i - x_3\partial\eta/\partial x_i$, where u_i is the displacement in the middle plane. Hence, (1.5) takes its final form

$$\frac{\partial\eta}{\partial t} - hH\frac{\partial}{\partial t}\nabla^2\eta - H\frac{\partial^2}{\partial t\,\partial x_i}\left(u_i + \frac{1}{3}\varphi_i\right) = 0 \tag{2.1}$$

Furthermore, the elastic stresses are defined by Hooke's law /5/

$$\frac{1-\nu^2}{E}\sigma_{ij}^{(1)} = (1-\nu)e_{ij} + \nu e\delta_{ij} - x_3\left[(1-\nu)\frac{\partial^2\eta}{\partial x_i\,\partial x_j} + \nu\nabla^2\eta\delta_{ij}\right] \tag{2.2}$$
$$e = e_{jj},\quad e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

where e_{ij} is the strain of the middle plane that is thereby related to the total stress in the lithosphere

$$N_{ij} = \frac{2hE}{1-\nu^2}\left[(1-\nu)e_{ij} + \nu e\delta_{ij}\right] \tag{2.3}$$

Here E is Young's modulus of the slab, and ν is its Poisson's ratio.

To find the contact stresses in (1.3) for the horizontal lithosphere motion $(k = 1)$, the continuity conditions should be used for the forces at the contact

$$\sigma_{13}^{(1)} = \sigma_{13}^{(2)} - (\sigma_{11}^{(1)} - \sigma_{11}^{(2)})\frac{\partial\eta}{\partial x_1} + (\sigma_{12}^{(1)} - \sigma_{12}^{(2)})\frac{\partial\eta}{\partial x_2} \tag{2.4}$$
$$\sigma_{23}^{(1)} = \sigma_{23}^{(2)} + (\sigma_{22}^{(1)} - \sigma_{22}^{(2)})\frac{\partial\eta}{\partial x_2} + (\sigma_{12}^{(1)} - \sigma_{12}^{(2)})\frac{\partial\eta}{\partial x_1},\quad \sigma_{33}^{(1)} = \sigma_{33}^{(2)}$$

and the analogous conditions on the boundary $x_3 = -h + \eta$

$$\sigma_{13}^{(1)} = (\sigma_{11}^{(1)} - Q_{11})\frac{\partial\eta}{\partial x_1} + \sigma_{12}^{(1)}\frac{\partial\eta}{\partial x_2} \tag{2.5}$$
$$\sigma_{23}^{(1)} = (\sigma_{22}^{(1)} - Q_{22})\frac{\partial\eta}{\partial x_2} + \sigma_{12}^{(1)}\frac{\partial\eta}{\partial x_1},\quad \sigma_{33}^{(1)} = -Q$$

where Q_{11}, Q_{22} are the overload components. Substituting (2.3)-(2.5) into (1.2), and taking the representations (1.7) and (2.2) into account, we obtain after linearization

$$\frac{2EhH}{(1+\nu)\mu}\left(\nabla^2 u_i + \frac{1+\nu}{1-\nu}\frac{\partial^2 u_j}{\partial x_i\,\partial x_j}\right) = -\left[(h-2H)\frac{\partial^2\eta}{\partial t\,\partial x_i} + \frac{\partial\varphi_i}{\partial t} - \frac{\partial u_i}{\partial t} + v_i^c - u_i^c\right] \tag{2.6}$$

where products of the type $[p(x_j, h-\eta, t)(Eh)]\,\partial\eta/\partial x_i$ have also been neglected.

To find the slab bending equations we use a well-known method /6/. We substitute (2.2) into the first two equilibrium Eqs.(1.1) for $k = 1$

$$\frac{\partial\sigma_{i3}^{(1)}}{\partial x_3} = \frac{Ex_3}{1-\nu^2}\frac{\partial}{\partial x_i}\nabla^2\eta - \frac{1}{2h}\left(\sigma_{i3}^{(1)}(h-\eta) - \sigma_{i3}^{(1)}(-h-\eta)\right). \tag{2.7}$$

Integrating (2.7) for a variable upper limit in x_3 enables the stresses themselves $\sigma_{i3}^{(1)}$ to be expressed in terms of the vertical displacement and the contact stress. Substituting the result in the third equation of (1.1) and integrating the latter with respect to x_3 between $-h+\eta$ and $h-\eta$ we obtain the bending equation

$$D\nabla^4\eta = 2\rho_1 gh + \sigma_{33}^{(1)}(h-\eta) - \sigma_{33}^{(1)}(-h-\eta) - \tag{2.8}$$
$$h\frac{\partial}{\partial x_i}\left[\sigma_{3i}^{(1)}(h-\eta) - \sigma_{3i}^{(1)}(-h-\eta)\right],\quad D = \frac{2}{3}\frac{Eh^3}{1-\nu^2}$$

Furthermore, by conditions (2.4) and (2.5) and the representations (1.2),(1.7),(2.2) and (2.3), both the viscous and the total horizontal forces can be introduced directly into (2.8). After some reduction, we obtain, neglecting squares of the perturbations of the variables in (2.8),

$$p_* \equiv p(x_j, h-\eta, t) = 2\rho_1 gh + Q - D\nabla^4\eta + \tag{2.9}$$
$$\mu h\left(3 - \frac{h}{2H}\right)\frac{\partial}{\partial t}\nabla^2\eta + N_{ij}^c\frac{\partial^2\eta}{\partial x_i\,\partial x_j} - h\frac{\partial}{\partial x_i}\left[(p_* - Q_{ii})\frac{\partial\eta}{\partial x_i}\right] -$$
$$\mu\left(2 - \frac{h}{2H}\right)\frac{\partial^2 u_i}{\partial t\,\partial x_i} - \frac{\mu h}{2H}\frac{\partial^2\varphi_i}{\partial t\,\partial x_i}$$

where $N_{ij}^c(2h)$ corresponds to the initial state of stress of the slab.

We integrate the third eqution in (1.1) and the continuity equation with respect to the variable x_3 for $k = 2$

$$p - p_* - \rho_2 g(x_3 - h - \eta) = \mu\int_{h-\eta}^{x_3}\nabla^2 v_3\,dx_3 + \mu\left(\frac{\partial v_3}{\partial x_3} - \frac{\partial v_3}{\partial x_3}\Big|_{h-\eta}\right) \tag{2.10}$$

$$v_3 - \frac{\partial \eta}{\partial t} = -\int_{h+\eta}^{x_3} \frac{\partial v_i}{\partial x_i} dx_3 \tag{2.11}$$

Substituting (2.11) into (2.10), taking account of (1.3), and integrating the result over the intensity of the liquid layer, we obtain the following relationship between the contact and mean pressures:

$$p_* = \langle p \rangle - \rho_2 g \left(H - \frac{3}{2}\eta\right) + \frac{\mu}{2H}\frac{\partial \eta}{\partial t} + \mu\left(h - \frac{7}{10}H\right)\frac{\partial}{\partial t}\nabla^2\eta - \frac{\mu h H^2}{5}\frac{\partial}{\partial t}\nabla^4\eta - \mu\frac{\partial^2 u_i}{\partial t\,\partial x_i} + \frac{\mu H^2}{5}\frac{\partial}{\partial t}\nabla^2\frac{\partial u_i}{\partial x_i} \tag{2.12}$$

On the basis of (2.12), the bending Eq.(2.9) is converted to the form

$$\langle p \rangle = q - \frac{3}{2}\rho_2 g\eta - \frac{\mu}{2H}\frac{\partial \eta}{\partial t} + \mu\left(2h + \frac{7}{10}H + \frac{h^2}{2H}\right)\frac{\partial}{\partial t}\nabla^2\eta + \frac{\mu h H^2}{5}\frac{\partial}{\partial t}\nabla^4\eta - D\nabla^4\eta - q_{ij}\frac{\partial^2\eta}{\partial x_i\,\partial x_j} - \mu\left(1 + \frac{h}{2H}\right)\frac{\partial^2 u_i}{\partial t\,\partial x_i} - \frac{\mu H^2}{5}\frac{\partial}{\partial t}\nabla^2\frac{\partial u_i}{\partial x_i} + \frac{\mu h}{2H}\frac{\partial^2 \varphi_i}{\partial t\,\partial x_i} \tag{2.13}$$

$$(q = 2\rho_1 g h + \rho_2 g H + Q,\ -q_{ij} = N_{ij}^c + h\,(2\rho_1 g h + Q)\,\delta_{ij} - Q_{ij})$$

Therefore, a closed system of equations of motion of the asthenospheric fluid (1.8),(2.1) and of the non-stationary strain Eqs.(2.6), (2.13) of the lithospheric slab for the six desired variables $\eta,\ u_i,\ \varphi_i,\ \langle p \rangle$ is obtained.

3. We introduce the wave potentials Φ and Ψ such that

$$u_1 = \frac{\partial \Phi}{\partial x_1} + \frac{\partial \Psi}{\partial x_2}, \quad u_2 = \frac{\partial \Phi}{\partial x_2} - \frac{\partial \Psi}{\partial x_1} \tag{3.1}$$

Then after eliminating the variables $u_i,\ \langle p \rangle$ the equations of motion mentioned are separated. The first equation is obtained from (2.6) and corresponds to the non-stationary redistribution of the rotational potential

$$(1-\nu)\varkappa\nabla^4\Psi - \frac{\partial}{\partial t}\nabla^2\Psi - \varepsilon_{3ij}\frac{\partial^2\varphi_i}{\partial t\,\partial x_j} + \varepsilon_{3ij}\left(\frac{\partial v_i^c}{\partial x_j} - \frac{\partial u_i^\circ}{\partial x_j}\right) = 0 \tag{3.2}$$

$$\varkappa = 2EhH/(\mu(1-\nu^2))$$

where ε_{3ij} is an alternating tensor. Applying the vortex operation to (1.8) and using relationship (3.2), we obtain

$$(1-\nu)\varkappa\left(\frac{H^2}{3}\nabla^6\Psi - \nabla^4\Psi\right) - \frac{\partial}{\partial t}\left(\frac{4H^2}{3}\nabla^4\Psi - \nabla^2\Psi\right) - \varepsilon_{3ij}\frac{\partial u_i^c}{\partial x_j} = 0 \tag{3.3}$$

It can therefore be seen that shear perturbations cannot result in a rise in the middle plane of the lithosphere; such motions are separated off from the bending. If free vibrations corresponding to (3.3) (without taking account of the stationary shear motion of the lithosphere) are sought in the form $\Psi = \Psi_* \exp\left[i\,(k_j x_j - \omega t)\right]$, we obtain the dispersion relation

$$\omega\frac{\mu}{G} = -\frac{4\,(3 - k^2H^2)\,k^2hH}{3 + 4k^2H^2}, \quad k^2 = k_1^2 + k_2^2 \tag{3.4}$$

which means that the shear tectonic wave is not a travelling wave: the initial perturbations damp out "on the spot" without translational shifts.

Eqs.(2.1) and (2.6) yield for the longitudinal wave potential Φ

$$\varkappa\nabla^4\Phi - 2\frac{\partial}{\partial t}\nabla^2\Phi + (2h + H)\frac{\partial}{\partial t}\nabla^2\eta + \frac{3}{2H}\frac{\partial \eta}{\partial t} = 0 \tag{3.5}$$

If the deflection η is identically zero, (2.6) reduces to the well-known Eq./1/ of tectonic waves for the simple horizontal compression-tension of the lithosphere

$$\frac{\partial^2\Phi}{\partial x^2} = \frac{\mu\,(1-\nu^2)}{EhH}\frac{\partial \Phi}{\partial t} \tag{3.6}$$

In the general case (3.5) should be supplemented by (2.1), which we will now write in the form

$$\frac{1}{3}\frac{\partial^2\varphi_i}{\partial t\,\partial x_i} = \frac{1}{H}\frac{\partial\eta}{\partial t} + h\frac{\partial}{\partial t}\nabla^2\eta - \frac{\partial}{\partial t}\nabla^2\Phi \tag{3.7}$$

as well as by the result of applying the divergence operation to the viscous asthenosphere flow Eq. (1.8). Taking account of (3.7) we write that equation in the form

$$3H\frac{\partial}{\partial t}\nabla^2\Phi - H(3h-H)\frac{\partial}{\partial t}\nabla^2\eta - 3\frac{\partial\eta}{\partial t} = \frac{2H^3}{\mu}\nabla^2\langle p\rangle \tag{3.8}$$

which in combination with (3.5) determines the effective system of two equations for the deflection η and the potential Φ.

4. We consider space-time longitudinal tectonic perturbations of the type $\Phi = \Phi_* \exp[i(k_jx_j - \omega t)]$, $\eta = \eta_* \exp[i(k_jx_j - \omega t)]$. Then taking account of (2.13) we reduce (3.5) and (3.8) to a homogeneous algebraic system

$$\begin{gathered}2Hk^2(\varkappa k^2 - 2i\omega)\Phi_* - i\omega[3 - 2H(2h + H)k^2]\eta_* = 0 \\ i\omega Hk^2[15 - 10H(H - 2h)k^2 + 2H^4k^4]\Phi_* + \{i\omega[15 - \\ 10H(3h - H)k^2 + H^2(20hH + 20h^2 + 7H^2)k^4 - \\ 2hH^5k^6] - 5(k^2/\mu)H^2Y\}\eta_* = 0 \\ Y = 2DHk^4 - 2Hq_{ij}k_ik_j + 3H\rho_2 g\end{gathered} \tag{4.1}$$

if q_{ij}, v_j^c, w_j^c are constants; summation is over the subscripts i, j. System (4.1) has a non-trivial solution if the following dispersion equation is satisfied:

$$\begin{gathered}(15 - 100H^2k^2 - 2H^4k^4 - 4H^6k^6)X^2 - \\ \frac{4hHk^2}{1-\nu^2}\Big[15 - 10(3h - H)Hk^2 + (20hH + 20h^2 + 7H^2)H^2k^4 - \\ 2hH^5k^6 - \frac{5(1-\nu^2)HY}{hE}\Big]X - 20hH^3k^4\frac{Y}{(1-\nu^2)E} = 0, \quad X = \frac{i\omega\mu}{E}\end{gathered} \tag{4.2}$$

It can be shown that for real values of the parameters of the "lithosphere + asthenosphere" system, the roots of this equation will correspond to long standing waves $\lambda \geqslant 2H$, which can be extended but not displaced translationally.

Indeed, for $\lambda \geqslant 2H$, $H \sim h$, $E \sim 10^{10}$ Pa, $3\rho_2 gH \sim 10^{10}$ Pa, $\nu^2 \sim 0{,}1$, $q_{ij} \sim 2\cdot 10^{13}$ Pa.m we have real roots, where $X > 0$ although it was assumed in the very derivation of the resultative equations that $\lambda \geqslant 2H$.

Another version of the analysis /7/ showed that for $\lambda < 2H$ the deduction concerning no translational displacement remains valid. The origin of travelling eaves is explained by the action of either a large initial horizontal force N_{ij} or constant asthenospheric flows apparently caused by a shift of the lithosphere.

5. The possibility of the existence of solitary travelling waves is investigated by searching for a solution dependent on the coordinate $\xi = n_jx_j - ct$, where c is the modulus of the velocity $c_i = cn_i$ of these waves. Replacement of the differentiation operations: $\partial/\partial t = -c\,d/d\xi$, $\partial/\partial x_i = n_i d/d\xi$ enables the system of Eqs. (1.8), (2.1), (2.6), (2.13) to be converted to the following system of ordinary differential equations

$$n_i\frac{d\varphi_i}{d\xi} = \frac{3}{H}(\eta - \eta_0) + 3h\frac{d^2\eta}{d\xi^2} - 3n_i\frac{du_i}{d\xi} \tag{5.1}$$

$$\begin{gathered}c\frac{d\varphi_i}{d\xi} = c\frac{du_i}{d\xi} - \varkappa\left[(1-\nu)\frac{d^2u_i}{d\xi^2} - (1-\nu)n_in_j\frac{d^2u_j}{d\xi^2}\right] - \\ c(h - 2H)n_i\frac{d^2\eta}{d\xi^2} - v_i^c - w_i^c\end{gathered} \tag{5.2}$$

$$\begin{gathered}v_i^c - c\frac{d\varphi_i}{d\xi} - cH^2\frac{d^3}{d\xi^3}\left(u_i - \frac{1}{3}\varphi_i\right) - \\ chH^2n_i\frac{d^4\eta}{d\xi^4} = -\frac{2H^2}{\mu}n_i\frac{d\langle p\rangle}{d\xi}\end{gathered} \tag{5.3}$$

$$\begin{gathered}\frac{d\langle p\rangle}{d\xi} = -\frac{3}{2}\rho_2 g\frac{d\eta}{d\xi} - \frac{c\mu}{2H}\left(\frac{3h}{H} - 1\right)\frac{d^2\eta}{d\xi^2} - \\ q_{ij}n_in_j\frac{d^3\eta}{d\xi^3} - c\mu\left(2h + \frac{7}{10}H + \frac{2}{H}h^2\right)\frac{d^4\eta}{d\xi^4} - D\frac{d^5\eta}{d\xi^5} - \\ \frac{c\mu H^2}{5}\frac{d^6\eta}{d\xi^6} - c\mu\left(1 + \frac{2h}{H}\right)\frac{d^2}{d\xi^2}\left(n_i\frac{du_i}{d\xi}\right) + \frac{c\mu H^2}{5}\frac{d^4}{d\xi^4}\left(n_i\frac{du_i}{d\xi}\right)\end{gathered} \tag{5.4}$$

where $\eta = \eta_0$ for $n_id\varphi_i/d\xi = 3hd^2\eta/d\xi^2 - 3n_idu_i/d\xi$. After eliminating the variable φ_i Eqs. (5.1), (5.2) reduce to the following

$$\frac{\varkappa}{2}\frac{d}{d\xi}\left(n_i\frac{du_i}{d\xi}\right) - cn_i\frac{du_i}{d\xi} = \frac{c}{2}(2h - H)\frac{d^2\eta}{d\xi^2} + \frac{3c}{4H}(\eta - \eta_0) - \frac{n_i}{4H}(v_i^c - w_i^c) \tag{5.5}$$

On the other hand, we eliminate φ_i and $\langle p\rangle$ from (5.3) by using (5.1) and (5.4)

$$3cn_i\frac{du_i}{d\xi} + 2cH(H+2h)\frac{d^2}{d\xi^2}\left(n_i\frac{du_i}{d\xi}\right) + \frac{2cH^4}{5}\frac{d^4}{d\xi^4}\left(n_i\frac{du_i}{d\xi}\right) - \tag{5.6}$$

$$\sum_{k=0}^{6} a_k\frac{d^k\eta}{d\xi^k} + n_i v_i^\circ - \frac{3c}{H}(\eta-\eta_0) = 0$$

$$a_0 = \frac{3c}{H}, \quad a_1 = \frac{3}{\mu}H^2\rho_2 g, \quad a_2 = 2c(3h-H)$$

$$a_3 = \frac{2}{\mu}H^2 q_{ij}n_i n_j, \quad a_4 = cH\left(4h^2 + 3hH + \frac{7}{5}H^2\right)$$

$$a_5 = \frac{2}{\mu}H^2 D, \quad a_6 = \frac{2c}{5}hH^4$$

Solving (5.5) for $n_i du_i/d\xi$ and substituting the result into (5.6), we obtain after some reduction

$$\left(1 + \frac{\varkappa}{2c}\frac{d}{d\xi}\right)\sum_{k=0}^{6} a_k\frac{d^k\eta}{d\xi^k} - c\sum_{k=0}^{6} b_k\frac{d^k\eta}{d\xi^k} + \frac{3c\eta_0}{4H} + \tag{5.7}$$

$$\frac{1}{4}n_i(v_i^\circ + 3w_i^\circ) = 0$$

$$b_0 = \frac{9}{4H}, \quad b_2 = 3(H+2h)$$

$$b_4 = H\,[(H+2h)^2 + {}^3/_{10}H^2], \ b_1 = b_3 = b_5 = 0, \ b_6 = {}^1/_5 H^4(H+2h)$$

The equation obtained can be solved in the form of a solitary travelling wave in the case when the inhomogeneous terms vanish. The corresponding condition results in the following expression for the wave velocity:

$$c_i = -\frac{H}{3\eta_0}(v_i^\circ + 3w_i^\circ) \tag{5.8}$$

It hence follows that for $v_i^\circ = 0$ the velocity vector of the solitary tectonic wave is collinear with the velocity vector w_i° of the stationary displacement of the lithosphere relative to the mesosphere if the deflection η_0 of the lithosphere that caused it is negative, if its bending to a free surface occurs. If $\eta_0 > 0$, the velocity c_i is opposite to w_i. Eq.(5.8) shows that $c \sim 10 - 100$ km/yr for physically justified values of the ratio $(v_i^\circ + 3w_i^\circ)/\eta_0$. The velocity c of the solitary wave thereby corresponds in order of magnitude to the velocity of a D-wave that is exposed in the distribution of the strongest local earthquakes, but its direction is not at all along the meridian here as is asserted in /8/.

Solutions of (5.7) are the sum of the exponential solutions $\eta_n = \eta_n^* \exp(k_n\xi)$, $n = 1, 2, \ldots, 7$, where k_n are the roots of the corresponding characteristic equation. For those for which the magnitude is estimated by the condition $k_n H < 1$, the equation (5.7) is replaced approximately by the following:

$$H\left(5c - \frac{3\varkappa\rho_2 gH}{2c\mu}\right)\frac{d^2\eta}{d\xi^2} - 3\left(\frac{\varkappa}{2H} - \frac{\rho_2 g}{\mu}H^2\right)\frac{d\eta}{d\xi} - \frac{3c}{4H}\eta = 0 \tag{5.9}$$

For the values $c \sim 30$ km/yr, $\mu/E \sim 3$ years the characteristic Eq.(5.9) yields the following roots: $k_{1,2} \sim 10^{-5}\,\mathrm{m}^{-1}$, $\lambda_1 \sim \lambda_2 \sim 100$ km. This means that the effective width of the solitary wave is of the order of 200 km. The general rule for the estimate is $\lambda \sim \mu c/E$, from which it follows, in particular, that as the asthenosphere viscosity μ decreases or the lithosphere stiffness G increases, the wavelength of the solitary wave decreases although its velocity remains unchanged.

In conformity with the theory being developed, solitary tectonic waves do not damp out because of the transfer of energy from the asthenospheric flow that compensates for the viscous dissipation. Therefore, the "lithosphere slab + asthenosphere flow" system is a "self-wave" system in the broad sense of this word /9/.

The authors are grateful to I.A. Garagash for discussing the research.

REFERENCES

1. KASAHARA K., Earthquake Mechanics. Cambridge Univ. Press, Cambridge, 1981.
2. NIKOLAYEVSKII V.N., Mechanics of geomaterials and earthquakes. Itogi Nauki i Tekhniki. Ser. Mekhanika Deformiruemogo tverdogo tela, Vol.15, VINITI, Moscow, 1983.
3. MALAMUD A.S. and NIKOLAYEVSKII V.N., Quantitative estimate of a tectonic cycle on mantle earthquakes of Gindukisha. Izv. Akad. Nauk TadzhikSSR. Otdel. Fiz.-Matem. Khim. Geol. Nauk, No.1 (91), (1984).
4. KAPITSA P.L., Wave flow of thin viscous fluid layers. Zh. Eksp. Teor. Fiz., Vol.18, No.1, 1948.
5. TIMOSHENKO S. and WOINOWSKY-KRIEGER S., Theory of Plates and Shells. McGraw-Hill, New York, 1959.

6. BREKHOVSKIKH L.M. and GONCHAROV V.V., Introduction to the Mechanics of Continuous Media, Nauka, Moscow, 1982.
7. NIKOLAYEVSKII V.N. and RAMAZANOV T.K., On waves of lithosphere interaction with the asthenosphere. Hydrogeodynamic Forerunners of Earthquakes. Nauka, Moscow, 1984.
8. GUBERMAN SH. A., *D*-waves and earthquakes. Computational Seismology. Theory and Analysis of Seismological Observations, No.12, Nauka, Moscow, 1979.
9. VASIL'YEV V.A., ROMANOVSKII YU.M. and YAKHNO V.G., Self-wave processes in distributed kinetic systems, Uspekhi Fiz. Nauk, Vol.128, No.4, 1979.

Translated by M.D.F.

PMM U.S.S.R.,Vol.49,No.3,pp. 362-366,1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

GENERALIZED DYNAMIC PROBLEM OF THERMOELASTICITY FOR A HALF-SPACE HEATED BY LASER RADIATION*

M.S. BOIKO

A generalized dynamic problem of thermoelasticity is solved for a half-space heated by laser radiation. Expressions for the displacements in the Rayleigh wave are obtained. The asymptotic form of the solution at a point at infinity is studied. It is shown that the magnitude of the displacements at the wave fronts depends essentially onthe value of the rate of propagation of heat.

1. Formulation of the problem. Let a beam of radiant energy fall, at the instant $\tau = 0$, on a circular region of a plane boundary of an elastic half-space. The position of every point of it is determined by the coordinates ρ, z, θ_1 of a cylindrical coordinate system. The radiation intensity volume density of the beam is

$$q_v(\rho, \tau) = q_1(\rho) H(\tau), \quad q_1(\rho) = \begin{cases} q_0, & 0 \leqslant \rho \leqslant R_0 \\ 0, & \rho > R_0 \end{cases} \tag{1.1}$$

($H(\tau)$ is Heaviside's function). We require to find the elastic stresses and displacements in the half-space when the radiant energy is absorbed. The variation in the temperature field caused by the deformation is ignorded.

The solution of this problem can be reduced to solving the following set of Eqs. /1/:

$$\left(\Delta - c_1^{-2}\frac{\partial^2}{\partial\tau^2}\right)\Phi = mt, \quad \left(\Delta - c_2^{-2}\frac{\partial^2}{\partial\tau^2}\right)\Psi = 0, \quad \left(\Delta - \frac{l}{a}\frac{\partial}{\partial\tau}\right)t = 0 \tag{1.2}$$
$$l = 1 - t_r\frac{\partial}{\partial\tau}, \quad m = \frac{3\lambda + 2\mu}{\lambda + 2\mu}\alpha_t$$

Here Φ, Ψ are the displacement potentials, t is temperature, c_1, c_2 are the velocities of the longitudinal and transverse wave, t_r is the thermal flux relaxation time, a is the thermal conductivity, λ, μ are the Lamè coefficients, α_t is the coefficient of thermal expansion, and Δ is the Laplace operator.

The solutions of the system must satisfy the following boundary and initial conditions:

$$\sigma_{zz} = \sigma_{\rho z} = 0, \quad -\lambda_q\frac{\partial t}{\partial z} = \eta l q_v \tag{1.3}$$

$$\Phi = \Psi = t = \frac{\partial t}{\partial\tau} = \frac{\partial\Phi}{\partial\tau} = \frac{\partial\Psi}{\partial\tau} = 0 \tag{1.4}$$

(σ_{ij} is the thermoelastic stress tensor, η is the absorption capacity and λ_q is the thermal conductivity.

2. Construction of the solution. We shall construct the solution of the problem using the contour-integral method /2/. Let us write the solution sought in the form of the Fourier-Bessel transform

*Prikl.Matem.Mekhan.,49,3,470-475,1985.